

## GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES COMMON FIXED POINT THEOREM IN INTUITIONISTIC FUZZY METRIC SPACE USING IMPLICIT RELATIONS

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### Abstract

The object of this paper is to use the concept of weak-compatible mapping and prove a common fixed point theorem in Intuitionistic Fuzzy Metric space using implicit relation. We have also cited an example in support of our result.

**Keywords and Phrases:** Intuitionistic Fuzzy Metric space, Common fixed points, Compatible maps and Weak compatibility. Compatible mappings o

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## I. INTRODUCTION

In 1965 the concept of fuzzy sets was defined by Zadeh [13]. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. As a generalization of fuzzy sets, Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets, In 2004, Park [9] defined the concept of intuitionistic fuzzy metric space with the help of continuous t- norms and continuous t-conorms. Recently, in 2006, Alacaet. al. [1] using the notion of intuitionistic fuzzy sets, defined the concept of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space which is introduced by Kramosil and Michalek [7]. Turkogluet. al. [11] gave generalization of Jungck's [5] common fixed point theorem in intuitionistic fuzzy metric spaces. They first created the concept of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces. The concept of weakly compatible mappings is most general as each pair of compatible mappings is weakly compatible but the converse is not true. After that, many authors proved common fixed point theorems using different mappings in such spaces. Al-Thagafi and N. Shahzad [2] introduced the concept of occasionally weakly compatible mappings which is more general than the concept of weakly compatible mappings. The aim of this paper is to use the concept of weak compatible mapping and prove a fixed point theorem in intuitionistic fuzzy metric space using implicit relations.

## II. PRELIMINARIES

**DEFINITION (2.1)[9]:** A binary operation  $*$ :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is continuous t-norm if  $*$  is satisfying the following conditions:

- (i)  $*$  is commutative and associative;
- (ii)  $*$  is continuous;
- (iii)  $a * 1 = a$  for all  $a \in [0, 1]$ ;
- (iv)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$ .

**DEFINITION (2.2)[9]:** A binary operation  $\diamond$ :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is continuous t-conorm if  $\diamond$  is satisfying the following conditions:

- (i)  $\diamond$  is commutative and associative;

- (ii)  $\diamond$  is continuous;  
 (iii)  $a \diamond 0 = a$  for all  $a \in [0, 1]$ ;  
 (iv)  $a \diamond b \geq c \diamond d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$ .

**DEFINITION (2.3)[1]:** A 5-tuple  $(X, M, N, *, \diamond)$  is said to be an intuitionistic fuzzy metric space if  $X$  is an arbitrary set,  $*$  is a continuous t-norm,  $\diamond$  is a continuous t-conorm and  $M, N$  are fuzzy sets on  $X^2 \times (0, \infty)$  satisfying the following conditions:-

- (i)  $M(x, y, t) + N(x, y, t) \leq 1$ , for all  $x, y \in X$  and  $t > 0$ ;  
 (ii)  $M(x, y, 0) = 0$ , for all  $x, y \in X$ ;  
 (iii)  $M(x, y, t) = 1$ , for all  $x, y \in X$  and  $t > 0$  if and only if  $x = y$ ;  
 (iv)  $M(x, y, t) = M(y, x, t)$ , for all  $x, y \in X$  and  $t > 0$ ;  
 (v)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ , for all  $x, y, z \in X$  and  $s, t > 0$ ;  
 (vi) For all  $x, y \in X$ ,  $M(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$  is continuous;  
 (vii)  $\lim_{t \rightarrow \infty} M(x, y, t) = 1$ , for all  $x, y \in X$  and  $t > 0$ ;  
 (viii)  $N(x, y, 0) = 1$  for all  $x, y \in X$ ;  
 (ix)  $N(x, y, t) = 0$  for all  $x, y \in X$  and  $t > 0$  if and only if  $x = y$ ;  
 (x)  $N(x, y, t) = N(y, x, t)$  for all  $x, y \in X$  and  $t > 0$ ;  
 (xi)  $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$  for all  $x, y, z \in X$  and  $s, t > 0$ ;  
 (xii) For all  $x, y \in X$ ,  $N(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$  is continuous;  
 (xiii)  $\lim_{t \rightarrow \infty} N(x, y, t) = 0$ , for all  $x, y$  in  $X$ .

Then  $(M, N)$  is called an intuitionistic fuzzy metric on  $X$ . The functions  $M(x, y, t)$  and  $N(x, y, t)$  denote the degree of nearness and the degree of non-nearness between  $x$  and  $y$  with respect to  $t$ , respectively.

**REMARK[2.1]:** Every fuzzy metric space  $(X, M, *)$  is an intuitionistic fuzzy metric space of the form  $(X, M, 1 - M, *, \diamond)$  such that t-norm  $*$  and t-conorm  $\diamond$  are associated as:  
 $x \diamond y = 1 - ((1-x) * (1-y))$ , for all  $x, y \in X$

**REMARK[2.2]:** In intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$ ,  $M(x, y, \cdot)$  is non-decreasing and  $N(x, y, \cdot)$  is non-increasing for all  $x, y \in X$ .

**EXAMPLE[2.1]:** Let  $(X, d)$  be a metric space, define t-norm  $a * b = \text{Min}\{a, b\}$  and t-conorm  $a \diamond b = \text{Max}\{a, b\}$  and for all  $x, y \in X$  and  $t > 0$

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then  $(X, M, N, *, \diamond)$  is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric  $(M, N)$  induced by the metric  $d$  the standard intuitionistic fuzzy metric.

**DEFINITION (2.4)[1]:** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space. Then

- (a) A sequence  $\{x_n\}$  in  $X$  is said to be Cauchy sequence if for all  $t > 0$  and  $p > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1, \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0$$

- (b) A Sequence  $\{x_n\}$  in  $X$  is said to be Convergent to a point  $x \in X$  if, for all  $t > 0$ ,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1, \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0$$

Since  $*$  and  $\diamond$  are continuous, the limit is uniquely determined from (v) and (xi) of definition (2.3), respectively.

**DEFINITION (2.5)[1]:** An intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be complete if and only if every Cauchy sequence in  $X$  is convergent.

**DEFINITION (2.6)[10]:** Let A and B be mappings from an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  into itself. Then the maps A and B are said to be compatible if, for all  $t > 0$ ,

$$\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1, \lim_{n \rightarrow \infty} N(ABx_n, BAx_n, t) = 0$$

whenever  $\{x_n\}$  is a sequence in X such that-  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = x$ , for some  $x \in X$ .

**DEFINITION (2.7)[6]:** Two self-maps A and B in a intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to be weak compatible if they commute at their coincidence points. i.e.  $Ax = Bx$  for some  $x$  in X, then  $ABx = BAx$ .

**DEFINITION (2.8)[6]:** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space, A and B be self maps in X, Then a point x in X is called a coincidence point of A and B iff  $Ax = Bx$ . In this case  $y = Ax = Bx$  is called a point of coincidence of A and B.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

**Lemma (2.1)[1]:** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space and  $\{y_n\}$  be a sequence in X, if there exist a number  $k \in (0, 1)$  such that

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$$

$$N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$$

for all  $t > 0$  and  $n=1, 2, 3, \dots$ , then  $\{y_n\}$  is a Cauchy sequence in X.

**Lemma (2.2)[12]:** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space and for all  $x, y$  in X,  $t > 0$  and if there exists a number  $k \in (0, 1)$

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t), \text{ then } x=y.$$

**Proposition 2.1.** Let  $\{x_n\}$  be a Cauchy sequence in intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  with continuous t-norm t. If the subsequence  $\{x_{2n}\}$  converges to x in X, then  $\{x_n\}$  also converges to x.

**A class of implicit relation.** Let  $\Phi$  be the set of all real continuous functions

$\phi : (R^+)^4 \rightarrow R$ , non-decreasing in the first argument with the property :

- a. For  $u, v \geq 0$ ,  $\phi(u, v, v, u) \geq 0$  or  $\phi(u, v, u, v) \geq 0$  implies that  $u \geq v$ .
- b.  $\phi(u, u, 1, 1) \geq 0$  implies that  $u \geq 1$ .

**Example 2.1.** Define:  $\phi(t_1, t_2, t_3, t_4) = 18t_1 - 16t_2 + 8t_3 - 10t_4$ ,  
 and  $\Psi(t_1, t_2, t_3, t_4) = 18t_1 - 16t_2 + 8t_3 - 10t_4$ . Then  $\phi, \Psi \in \Phi$ .

### III. MAIN RESULT

**Theorem 3.1.** Let A, B, S and T be self-mappings on an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  with continuous t-norm and continuous t-conorm satisfying:

- (3.1.1)  $A(X) \subseteq S(X), B(X) \subseteq T(X);$
- (3.1.2) One of  $S(X), B(X), T(X)$  or  $A(X)$  is complete;
- (3.1.3) The pairs (A, T) and (B, S) are weak-compatible;
- (3.1.4) for some  $\phi, \Psi \in \Phi$ , there exists  $k \in (0, 1)$  such that for all  $x, y \in X$  and  $t > 0$ ,  
 $\phi( M(Ax, By, kt), M(Tx, Sy, t), M(Ax, Tx, t), M(By, Sy, kt) ) \geq 0$   
 and  $\Psi( N(Ax, By, kt), N(Tx, Sy, t), N(Ax, Tx, t), N(By, Sy, kt) ) \leq 0$

then A, B, S and T have a unique common fixed point in X.

**Proof.** Let  $x_0 \in X$ . From condition (3.1.1),  $\exists x_1, x_2 \in X$  such that

$$Ax_0 = Sx_1 = y_0 \quad \text{and} \quad Bx_1 = Tx_2 = y_1.$$

Inductively, we can construct sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$$Ax_{2n} = Sx_{2n+1} = y_{2n} \quad \text{and} \quad Bx_{2n+1} = Tx_{2n+2} = y_{2n+1}$$

for  $n = 0, 1, 2, \dots$

**Step I.** Putting  $x = x_{2n}$  and  $y = x_{2n+1}$  in (3.1.4), we get

$$\phi(M(Ax_{2n}, Bx_{2n+1}, kt), M(Tx_{2n}, Sx_{2n+1}, t), M(Ax_{2n}, Tx_{2n}, t), M(Bx_{2n+1}, Sx_{2n+1}, kt)) \geq 0$$

and

$$\Psi(N(Ax_{2n}, Bx_{2n+1}, kt), N(Tx_{2n}, Sx_{2n+1}, t), N(Ax_{2n}, Tx_{2n}, t), N(Bx_{2n+1}, Sx_{2n+1}, kt)) \leq 0.$$

Letting  $n \rightarrow \infty$ , we get

$$\phi(M(y_{2n}, y_{2n+1}, kt), M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n-1}, t), M((y_{2n+1}, y_{2n}, kt))) \geq 0$$

$$\Psi(N(y_{2n}, y_{2n+1}, kt), N(y_{2n-1}, y_{2n}, t), N(y_{2n}, y_{2n-1}, t), N((y_{2n+1}, y_{2n}, kt))) \leq 0.$$

Using (a), we get

$$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t) \text{ and } N(y_{2n}, y_{2n+1}, kt) \leq N(y_{2n-1}, y_{2n}, t).$$

Therefore, for all  $n$  even or odd, we have

$$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t) \text{ and } N(y_n, y_{n+1}, kt) \leq N(y_{n-1}, y_n, t).$$

Therefore, by lemma 2.1,  $\{y_n\}$  is a Cauchy sequence in  $X$ .

**Case I.**  $S(X)$  is complete.

In this case  $\{y_{2n}\} = \{Sx_{2n+1}\}$  is a Cauchy sequence in  $S(X)$ , which is complete. Thus  $\{y_{2n}\}$  converges to some  $z \in S(X)$ .

By Proposition 2.1, we have

$$\begin{aligned} \{Bx_{2n+1}\} \rightarrow z & \quad \text{and} \quad \{Sx_{2n+1}\} \rightarrow z \\ \{Ax_{2n}\} \rightarrow z & \quad \text{and} \quad \{Tx_{2n}\} \rightarrow z. \end{aligned}$$

As  $z \in S(X)$  there exists  $u \in X$  such that  $z = Su$ :

**Step I.** Putting  $x = x_{2n}$  and  $y = u$  in (3.1.4) we get,

$$\phi(M(Ax_{2n}, Bu, kt), M(Tx_{2n}, Su, t), M(Ax_{2n}, Tx_{2n}, t), M(Bu, Su, kt)) \geq 0$$

$$\Psi(N(Ax_{2n}, Bu, kt), N(Tx_{2n}, Su, t), N(Ax_{2n}, Tx_{2n}, t), N(Bu, Su, kt)) \leq 0.$$

Taking limit as  $n \rightarrow \infty$ , we get

$$\phi(M(z, Bu, kt), M(z, z, t), M(z, z, t), M(Bu, z, kt)) \geq 0 \quad \text{and}$$

$$\Psi(N(z, Bu, kt), N(z, z, t), N(z, z, t), N(Bu, z, kt)) \leq 0$$

$\Rightarrow \phi(M(z, Bu, kt), 1, 1, M(Bu, z, kt)) \geq 0$  and

$$\Psi(N(z, Bu, kt), 1, 1, N(Bu, z, kt)) \leq 0.$$

Using (b) we have

$$M(z, Bu, kt) \geq 1, \text{ for all } t > 0.$$

$$N(z, Bu, kt) \leq 0, \text{ for all } t > 0$$

Hence  $M(z, Bu, t) = 1$  and  $N(z, Bu, t) = 0$

Hence  $Su = Bu = z$ , as  $(BS)$  is weak compatible so we have

$$Bz = Sz = z.$$

**Step II.** Putting  $x = x_{2n}$  and  $y = z$  in (3.1.4) we get,

$$\phi(M(Ax_{2n}, Bz, kt), M(Tx_{2n}, Sz, t), M(Ax_{2n}, Tx_{2n}, t), M(Bz, Sz, kt)) \geq 0 \quad \text{and}$$

$$\Psi(N(Ax_{2n}, Bz, kt), N(Tx_{2n}, Sz, t), N(Ax_{2n}, Tx_{2n}, t), N(Bz, Sz, kt)) \leq 0.$$

Taking limit as  $n \rightarrow \infty$ , we get

$$\phi(M(z, Bz, kt), M(z, z, t), M(z, z, t), M(Bz, z, kt)) \geq 0 \quad \text{and}$$

$$\Psi(N(z, Bz, kt), N(z, z, t), N(z, z, t), N(Bz, z, kt)) \leq 0.$$

$$\Rightarrow \phi(M(z, Bz, kt), 1, 1, M(Bz, z, kt)) \geq 0 \text{ and}$$

$$\Psi(N(z, Bz, kt), 1, 1, N(Bz, z, kt)) \leq 0$$

Using (b) we have

$$M(z, Bz, kt) \geq 1, \text{ for all } t > 0.$$

$$N(z, Bz, kt) \leq 0, \text{ for all } t > 0.$$

Hence

$$M(z, Bz, t) = 1 \quad \text{and} \quad N(z, Bz, t) = 0$$

Thus  $z = Bz$ .

**Step III.** As  $B(X) \subseteq T(X)$  there exists  $v \in X$  such that

$$z = Bz = Tv.$$

Putting  $x = v$  and  $y = x_{2n+1}$  in (3.1.4), we get

$$\phi(M(Av, Bx_{2n+1}, kt), M(Tv, Sx_{2n+1}, t), M(Av, Tv, t), M(Bx_{2n+1}, Sx_{2n+1}, kt)) \geq 0$$

and

$$\Psi(N(Av, Bx_{2n+1}, kt), N(Tv, Sx_{2n+1}, t), N(Av, Tv, t), N(Bx_{2n+1}, Sx_{2n+1}, kt)) \leq 0.$$

Taking limit as  $n \rightarrow \infty$ , we get

$$\phi(M(Av, z, kt), M(z, z, t), M(Av, z, t), M(z, z, kt)) \geq 0 \text{ and}$$

$$\Psi(N(Av, z, kt), N(z, z, t), N(Av, z, t), N(z, z, kt)) \leq 0$$

$$\Rightarrow \phi(M(Av, z, kt), 1, M(Av, z, t), 1) \geq 0 \text{ and}$$

$$\Psi(N(Av, z, kt), 1, N(Av, z, t), 1) \leq 0.$$

As  $\phi$  is non-decreasing and  $\Psi$  is non-increasing in the first argument, we have

$$\Rightarrow \phi(M(Av, z, t), 1, M(Av, z, t), 1) \geq 0 \text{ and}$$

$$\Psi(N(Av, z, t), 1, N(Av, z, t), 1) \leq 0$$

Using (a) we have

$$M(Av, z, t) \geq 1, \text{ for all } t > 0.$$

$$N(Av, z, t) \leq 0, \text{ for all } t > 0.$$

Hence

$$M(Av, z, t) = 1 \quad \text{and} \quad N(Av, z, t) = 0$$

Thus  $Av = z$ , therefore  $Bz = Az = Tz$ .

**Step IV.** Putting  $x = z$  and  $y = x_{2n+1}$  in (3.1.4), we get

$$\phi(M(Az, Bx_{2n+1}, kt), M(Tz, Sx_{2n+1}, t), M(Az, Tz, t), M(Bx_{2n+1}, Sx_{2n+1}, kt)) \geq 0$$

and

$$\Psi(N(Az, Bx_{2n+1}, kt), N(Tz, Sx_{2n+1}, t), N(Az, Tz, t), N(Bx_{2n+1}, Sx_{2n+1}, kt)) \leq 0.$$

Letting  $n \rightarrow \infty$ , we get

$$\phi(M(Az, z, kt), M(Az, z, t), M(Az, Az, t), M(z, z, kt)) \geq 0 \text{ and}$$

$$\Psi(N(Az, z, kt), N(Az, z, t), N(Az, Az, t), N(z, z, kt)) \leq 0$$

$$\Rightarrow \phi(M(Az, z, kt), M(Az, z, t), 1, 1) \geq 0 \text{ and}$$

$$\Psi(N(Az, z, kt), N(Az, z, t), 1, 1) \leq 0.$$

As  $\phi$  is non-decreasing and  $\Psi$  is non-increasing in the first argument, we have

$$\Rightarrow \phi(M(Az, z, t), M(Az, z, t), 1, 1) \geq 0 \text{ and}$$

$$\Psi(N(Az, z, t), N(Az, z, t), 1, 1) \leq 0$$

Using (b), we have

$$M(Az, z, t) \geq 1, \text{ for all } t > 0$$

$$N(Az, z, t) \leq 0, \text{ for all } t > 0.$$

Hence

$$M(Az, z, t) = 1 \quad \text{and} \quad N(Az, z, t) = 0.$$

Thus  $Az = z$ .

Combining the results from different steps, we have

$$Az = Bz = Tz = Sz = z.$$

Hence the four self-maps have a common fixed point in this case.

Case when  $A(X)$  is complete follows from above case as  $A(X) \subseteq S(X)$ .

**Case II.**  $T(X)$  is complete. This case follows by symmetry.

As  $B(X) \subseteq T(X)$ , therefore the result also holds when  $B(X)$  is complete.

**Uniqueness.** Let  $u$  be another common fixed point of  $A, B, S$  and  $T$ , then

$$Au = Bu = Su = Tu = u.$$

Putting  $x = z$  and  $y = w$  in (3.1.4), we get

$$\phi(M(Az, Bw, kt), M(Tz, Sw, t), M(Az, Tz, t), M(Bw, Sw, kt)) \geq 0 \quad \text{and} \\ \Psi(N(Az, Bw, kt), N(Tz, Sw, t), N(Az, Tz, t), N(Bw, Sw, kt)) \leq 0$$

$$\Rightarrow \phi(M(z, w, kt), M(z, w, t), M(z, z, t), M(w, w, kt)) \geq 0 \quad \text{and} \\ \Psi(N(z, w, kt), N(z, w, t), N(z, z, t), N(w, w, kt)) \leq 0$$

$$\Rightarrow \phi(M(z, w, kt), M(z, w, t), 1, 1) \geq 0 \quad \text{and} \\ \Psi(N(z, w, kt), N(z, w, t), 1, 1) \leq 0.$$

As  $\phi$  is non-decreasing and  $\Psi$  is non-increasing in the first argument, we have

$$\phi(M(z, w, t), M(z, w, t), 1, 1) \geq 0 \quad \text{and} \\ \Psi(N(z, w, t), N(z, w, t), 1, 1) \leq 0.$$

Using (b), we have

$$M(z, w, t) \geq 1, \quad \text{for all } t > 0. \\ N(z, w, t) \leq 0, \quad \text{for all } t > 0.$$

Hence

$$M(z, w, t) = 1 \quad \text{and} \quad N(z, w, t) = 0$$

Thus  $z = w$ .

Therefore,  $z$  is a unique common fixed point of  $A, B, S$  and  $T$ .

This completes the proof.

**Example 3.1:** Let  $(X, d)$  be a Metric space, where  $X = [0, 1]$  and  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space, Define self-maps  $L, M, A$  and  $S$  as follows:

$$L(X) = M(X) = \begin{cases} 0, & x \in [0, \frac{5}{6}] \\ 1 - x, & \text{otherwise} \end{cases},$$

$$A(X) = \begin{cases} 0, & x \in [0, \frac{4}{5}] \\ 1 - x, & \text{otherwise} \end{cases}$$

$$S(X) = \begin{cases} 0, & x \in [0, \frac{3}{4}] \\ 1 - x, & \text{otherwise} \end{cases}.$$

Then  $L(X) = M(X) = [0, \frac{1}{6}]$ ,  $A(X) = [0, \frac{1}{5}]$  and  $S(X) = [0, \frac{1}{4}]$ , Hence the pair  $(L, A)$  and  $(M, A)$  are weak compatible and  $A(X)$  is complete. Further, for  $k = \frac{1}{3}$  the condition (3.1.3) is satisfied. Thus, 0 is the unique common fixed point of the mappings  $A, L, M$ , and  $S$ .

#### IV. CONCLUSION

In view of theorem 3.1 is generalization of the result. And prove common fixed point theorem for four self-mappings in intuitionistic fuzzy metric space using implicit relations.

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